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Effects of Jitter on AD/DA conversion. Specification of Clock Jitter Performance.

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ABSTRACT

The impact of clock jitter on AD/DA conversion performance is detailed for several conversion methods. Account is taken of the spectral distribution of both the jitter and of the converted waveform. The inadequacy of a single "pico-second" performance figure is shown, and the use of a dBc/sqrt(Hz) specification is proposed instead.

1. INTRODUCTION

Clock jitter has become known among designers and users of digital audio equipment as a source of performance degradation. Despite this recognition, unambiguous specifications and performance standards are not commonplace. Although a traditional period jitter specification is sufficient to verify if data integrity will be preserved, it does little to predict how well AD/DA converters will operate when driven by a particular clock. It is precisely this performance issue that the present paper will look at, with the goal of finding a specification that does link up with analogue audio performance.

2. DEFINITIONS

f_c	Clock frequency
M(t)	Instantaneous value of jittery clock signal, amplitude normalised (peak=1, DC=0)
J(t)	Instantaneous value of phase error, angle.
ω_c	Clock pulsation ($2\cdot\pi\cdot f_c$)
ω_m	Pulsation of sinusoidal modulation
$t_{j,pk}$	Amplitude of phase deviation expressed as time, peak value
φ_{pk}	Amplitude of phase deviation expressed as angle, peak value
$t_{j,rms}$	Amplitude of phase deviation expressed as time, RMS value
$t_{j,rms,1Hz}(f)$	Phase deviation caused by all jitter components in an infinitesimal band around f, normalised to 1Hz.
$t_{j,rms,f1,f2}$	Integrated phase deviation caused by all

	jitter components between f1 and f2.
D	Amplitude relative to carrier of one modulation sideband caused by a given jitter component. May be expressed in dBc.
d(f)	Amplitude relative to carrier of one side band caused by all jitter components in an infinitesimal band around f, normalised to 1Hz. May be expressed in dBc.
f_{max}	Audio bandwidth limit (e.g. 20kHz)
f_s	Converter sampling rate (includes upsampling) at the z to s domain boudary.
SFDR	Spurious-free dynamic range. Ratio between the maximum amplitude of the converter and the highest single unwanted (spurious) component.
SNR	Signal-to-noise ratio.

3. BASIC CONCEPTS

3.1. Phase modulation

Jitter is defined as phase modulation of a clock of pulsation ω_C by a signal J(t). We shall limit ourselves to the first harmonic:

$$M(t) = \sin(\omega_c \cdot t + J(t))$$

3.2. Spectral makeup

Presuming the modulation index (phase deviation) is low, the frequency domain representation of the resulting signal is that of a carrier flanked by a pair of mirror-image side bands, each the image of the modulating signal. Higher order modulation components are negligible (Armstrong modulation).

Assuming M(t) a sinusoidal signal of amplitude φ_{pk} and frequency ω_m.

$$M(t) = \sin(\omega_c \cdot t + \varphi_{pk} \cdot \sin(\omega_m \cdot t)) \approx \sin(\omega_c \cdot t) + \frac{\varphi_{pk}}{2} \cdot (\sin((\omega_c + \omega_m) \cdot t) - \sin((\omega_c - \omega_m) \cdot t))$$

A result of this is that jitter can be expressed not only as time (phase), but also as the ratio D of sideband amplitude to carrier amplitude.

$$D = \frac{\varphi_{pk}}{2} = \frac{t_{j,pk} \cdot \omega_c}{2} = t_{j,pk} \cdot f_c \cdot \pi = t_{j,rms} \cdot f_c \cdot \pi \cdot \sqrt{2}$$

This ratio D can then be expressed in dBc (dB relative to carrier).

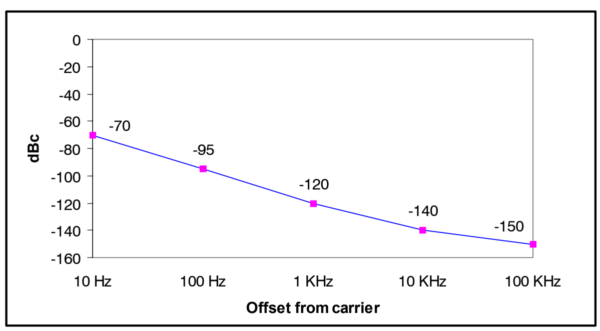
$$D = 20 \cdot \log\left(t_{j,rms} \cdot f_c \cdot \pi \cdot \sqrt{2}\right) \text{ (dBc)}$$

The big advantage of using this method is that the number can be read straight off a spectrum analyser display.

3.3. Phase noise

Clock jitter is sometimes called “phase noise”. Noise, since the modulation is unwanted, and since it is normally of a wideband, random nature. In this case, the modulation is not a single sinusoidal component, but an infinite number of them. In a similar vein to other types of random noise, they are treated as power per unit bandwidth or as linear units per root of unit bandwidth. However, somewhat counterintuitively phase noise is often converted from linear units to decibels, producing an abstract unit referred to in shorthand (but rather uncorrectly) as dBc per root Hertz. This is the accepted practice in the telecomms industry.

Data sheets of *real* low-noise oscillators (such as those used in time bases or frequency standards) normally sport a graph or table of phase noise versus offset (difference) frequency.



This graph was taken from an above-average quality oscillator. It shows that jitter rises quite rapidly at decreasing frequency offsets.

Alternatively, the phase noise can be specified in seconds per root Hz (the reader is pleaded not to refer to this unit as $s^{1.5}$)

$$t_{j,rms,1Hz}(f) = \frac{10^{\frac{d(f)}{20}}}{f_c \cdot \pi \cdot \sqrt{2}} \quad (s/Hz^{0.5})$$

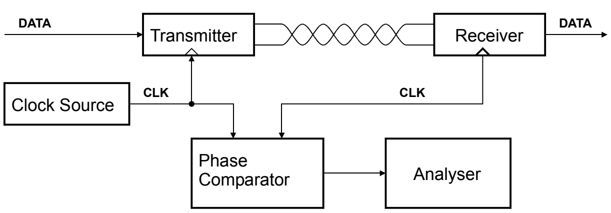
Where d(f) is the phase noise density quoted in dBc/sqrt(Hz) at frequency f. As we shall find, s/sqrt(Hz) is quite practical for calculating converter performance. The amount of jitter found over a range f1 to f2, expressed in seconds, is found by integration.

$$t_{j,rms,f1,f2} = \sqrt{\int_{f1}^{f2} t_{j,rms,1Hz}(f)df}$$

4. INTERFACE VERSUS CLOCK JITTER

4.1. Interface jitter

When data and clock signals are transmitted in multiplex, as in AES3 links, interface jitter is found as the variation in the delay between an edge on the transmitter's clock and the corresponding edge of the received clock. Since the two clocks are locked in the long term, the absolute maximum timing error will be limited. The source clock does not even need to have a particularly good long-term stability in order to measure interface jitter reliably.



For this reason, it is customary to specify the jitter of data interface circuits in terms of an RMS or peak-to-peak value, and to quote this as one number. While this number gives nothing away about the spectral composition of the jitter, at least it has physical meaning.

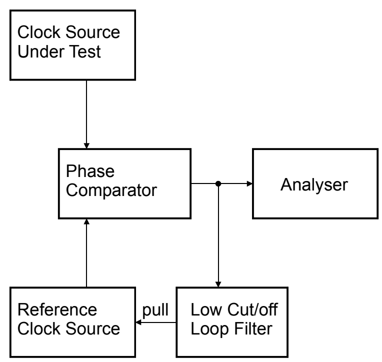
Interface jitter is typically a mix of largely white noise, some regular, discrete components and data-related components. This paper concentrates on random jitter. The study of regular jitter patterns is postponed to a later instalment.

4.2. Clock jitter

Unfortunately, the practice of spec'ing a single figure of merit has also become commonplace in relation to clock oscillators in audio – more precisely in statements such as "the clock oscillator in this converter has less than 5ps jitter" and "jitter below 50ps has been shown to be audible".

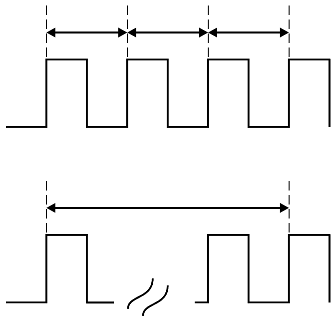
According to 3.2, it would be sufficient to feed the oscillator output to a spectrum analyser and simply read the output spectrum from the carrier upward. For very jittery signals this is certainly an option. One would find, however, that a good quality clock oscillator will stretch even the best analyser beyond its limits.

Clock jitter measurements are normally carried out using a pullable "reference quality" source and a phase detector, wired together to form a very slow PLL.



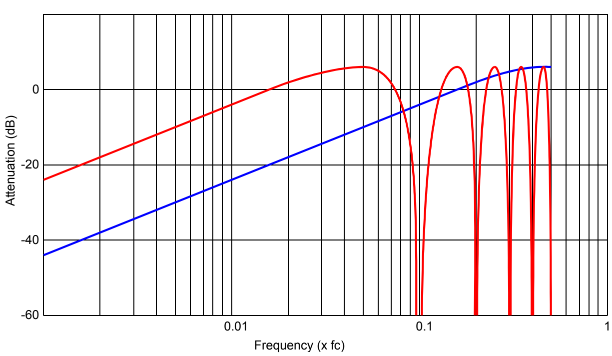
Hopefully, the oscillator under test is sufficiently stable to keep the phase difference at the detector within +/-180º. Otherwise, the PLL cutoff frequency must be increased, rendering the results unreliable for low frequency offsets. The output of the phase detector can be considered to be a direct measure of instantaneous deviation of the DUT from the "ideal". To read the noise amplitude directly as an RMS value is rarely done – this test will produce any desired peak jitter number, simply by extending the measurement bandwidth downward.

Several single-number jitter specs do exist though. The two most common ones are period jitter and long-term jitter.



Period jitter (top) measures the times between two successive corresponding edges. It is a differential measurement and correspondingly weighs the noise spectrum with a 20dB/dec upward slope. The unity gain point is at $f_c/2\pi$. Given that most of the jitter occurs at low frequencies, period jitter is a rather optimistic number. It is also the most oft-quoted number in audio applications. It is now easy to see how a "50ps" analyser reading can hide a jitter problem that is demonstrably audible.

Long-term jitter (bottom) is similar to period jitter, but it measures across a large number n of periods. Hundreds of thousands of periods are common. In this way, gain at low frequencies is increased by a factor n. At high frequencies, we find the function is actually a comb (sinusoidal) function.



Shown above is the weighting function (difference compared to a reference-based measurement) implicitly applied by the period jitter (n=1) and long-term jitter (n=10) procedures.

5. EFFECTS OF JITTER ON AD/DA CONVERTERS

Whether caused by the clock circuit or by an interface, jitter that finds its way into a converter will proceed to negatively affect performance there, by effecting phase modulation on every signal component present on the boundary between z and s domains.

The general analysis consists of considering a sinusoidally jittered (phase-modulated) clock, mentally sweeping the modulation frequency from 0 to $f_c/2$. Every component that comes within f_{max} of the swept tone will produce an intermodulation component inside the band between DC and f_{max}. Conversely, to keep this from happening, at frequencies containing significant energy, the clock should have minimal jitter.

The reasoning of 3.2, holds equally well for an audio signal that is being modulated by jitter.

$$D = t_{j,rms} \cdot f \cdot \pi \cdot \sqrt{2}$$

The analysis will presume jitter is random (though not white), allowing power averaging. Nonrandom and possibly signal-dependent jitter should be analysed on a case by case basis, or better, avoided altogether.

5.1. Non-noise-shaped multibit converters

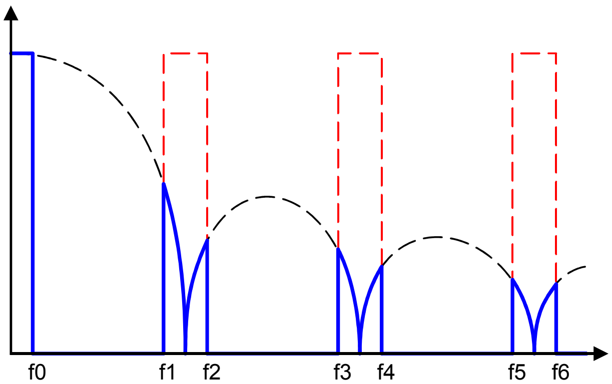
This is the fundamental case. Only signal-related components are present: the audio signal and its HF aliases. There is no other noise present, so SNR is not affected by jitter. The worst case condition for jitter detection is the application of a full-scale sine wave at f_{max}.

f_0	f_{max}
f_1	f_s-f_{max}
f_2	f_s+f_{max}
f_3	$2\cdot f_s$-f_{max}
f_4	$2\cdot f_s$+f_{max}
f_n	&c. ad $n=f_c/f_s$

A DAC will have a zero-order hold function that modifies the spectrum by a sinc function:

$$k_n = \mathrm{sinc}\left(\frac{\pi \cdot f_n}{f_s}\right)$$

This mostly affects the aliases ($k_0 \approx 1$). ADCs do not have this effect, making them somewhat more sensitive to jitter at higher frequencies.



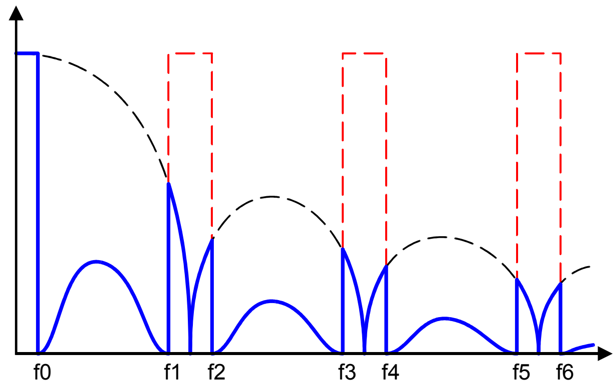
Sensitive areas are DC to $2 \cdot f_{max}$ and all its aliases. Total noise level relative to the test signal at fmax is found by integrating noise power across each band and summing all of these.

$$N_A = \pi \cdot \sqrt{0.5 \cdot \sum_{n=0}^{f_c/f_s - 1} f_n^2 \cdot k_n^2 \cdot \int_{f_n - f_{max}}^{f_n + f_{max}} t_{j,rms,1Hz}^2(f)df}$$

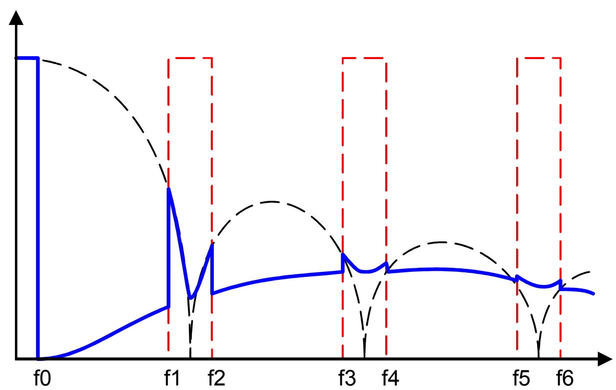
Integration at n=0 poses a question: jitter of even the best oscillators becomes large at very low frequencies ("close in"). When should we count the sidebands as "noise" and when is it part of the "legitimate" signal? Where does the carrier end and where does jitter begin? The question is nearly a philosophical one that only listening tests can settle. For now, we should content ourselves with the measurement capability of the analyser, ie. the width of the notch placed at f_{max}.

5.2. Noise shaped multibit converters

In noise shaped converters, the signal and its aliases are accompanied by another signal: shaped noise. For the scope of this paper, the noise is random enough to be thus simplified. Depending on the particular architecture, the noise shaper may or may not operate at the same sample frequency as the upsampling filter.



More often than not, the noise shaper runs substantially faster than he upsampling filter. In this case, the sinc function is applied to the noise spectrum separately and scaled corresponding to its sample rate (shown below is a situation where the noise shaper runs four times faster than the filter).



The noise contribution by the jitter breaks down into two parts. One is associated with demodulating aliases and is analysed as 5.1. The other part comes with the shaped noise. If the SNR requirement of a converter is more stringent than THD+N, the latter is considered alone.

To analyse at length the precise shape of the quantisation noise would be beyond the scope of this work. For the purpose of making good estimates, the noise can simply be said to be constant from DC upward to $f_s/2$, and to have a level typically 6dB above the quantisation noise of the quantiser alone. The band over which quantisation noise is actually reduced is negligible for this analysis. Let w be the word length of the quantiser. k(f) is the noise content per unit bandwidth relative to maximum modulation. An approximation of the quantisation noise level (expressed in 1/sqrt(Hz)) is:

$$k(f) = \frac{2}{2^w \cdot \sqrt{f_s/2}}$$

for

$$n \cdot f_s + f_{max} < f < (n+1) \cdot f_s - f_{max}$$

$$k(f) = 0 \text{ otherwise}$$

Then, integrated noise becomes

$$N_Q = \pi \cdot \sqrt{0.5 \cdot \sum_{n=0}^{\frac{f_c}{2 \cdot f_s} - 1} \int_{n \cdot f_s + f_{max}}^{(n+1) - f_{max}} f_Q^2 \cdot k^2(f_Q) \cdot \mathrm{sinc}\left(\frac{\pi \cdot f_Q}{f_s}\right) \cdot \int_{f_Q - f_{max}}^{f_Q + f_{max}} t_{j,rms,1Hz}^2(f) \cdot df \cdot df_Q}$$

Otherwise put, for every frequency in the range where quantisation noise is present (=everywhere except the aliases of the audio band around f_c), all intermodulation products with a band +/-f_{max} around this frequency are integrated.

5.3. Noise shaped converters with switched-capacitor filters

These converters are actually the most common type today. A large portion, if not all, of the reconstruction filter is done in the same z-domain as the conversion. This filter itself is totally oblivious to any jitter that might be present on the clock. Only the noise still present in the filtered output signal will demodulate clock jitter. This is why these converters are generally known as having low jitter sensitivity.

Analysis is similar to that in 5.2, except that the noise function k(f) should include the transfer function of the switched-capacitor filter. The same should be done for the aliases under 5.1.

5.4. 1-bit Deltasigma converters

Deltasigma converters analyse similarly to other noise-shaped converters except that noise gain is around 3dB and that a dominant tone is present.

The dominant tone is a discrete tone which nominally sits at $f_s/2$, and which is recognisable as frequent bursts of “10101010” bit patterns. It decreases linearly with modulation index (regardless of sign). In terms of jitter demodulation, this tone is an annoying presence. If any jitter is induced on the clock at $f_s/2$ (a bit clock or so), it will demodulate with the dominant tone as it wobbles around at low modulation indexes.

5.5. PWM converters

PWM converters are different from other noise shaped converters in that they have a full-scale carrier present at the sampling rate. It and its odd harmonics need to be taken into account when determining the maximum SNR attainable when a particular clock is used.

$$N_S = \pi \cdot \sqrt{0.5 \cdot \sum_{n=0}^{\frac{f_c}{2 \cdot f_s} - 2} f_s^2 \cdot \int_{(2 \cdot n - 1) \cdot f_s - f_{max}}^{(2 \cdot n - 1) \cdot f_s + f_{max}} t_{j,rms,1Hz}^2(f) df}$$

6. CONCLUSIONS

The effect of random jitter on different types of AD/DA converters has been shown and calculations given. It was shown that the use of a jitter specification in seconds per root hertz is the most practical for use in such calculations. dBc/sqrt(Hz) phase noise specifications as already used in other industries are easily converted into the s/sqrt(Hz) format. It is proposed to adopt dBc/sqrt(Hz) as a common practice in specifying clock oscillator performance for digital audio.

7. REFERENCES

[1] Dunn, Julian, “Jitter: Specification and Assessment in Digital Audio Equipment”, presented at the 93rd AES Convention, 1992.