Low Frequency Sound Generation by Loudspeaker Drivers

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Contents

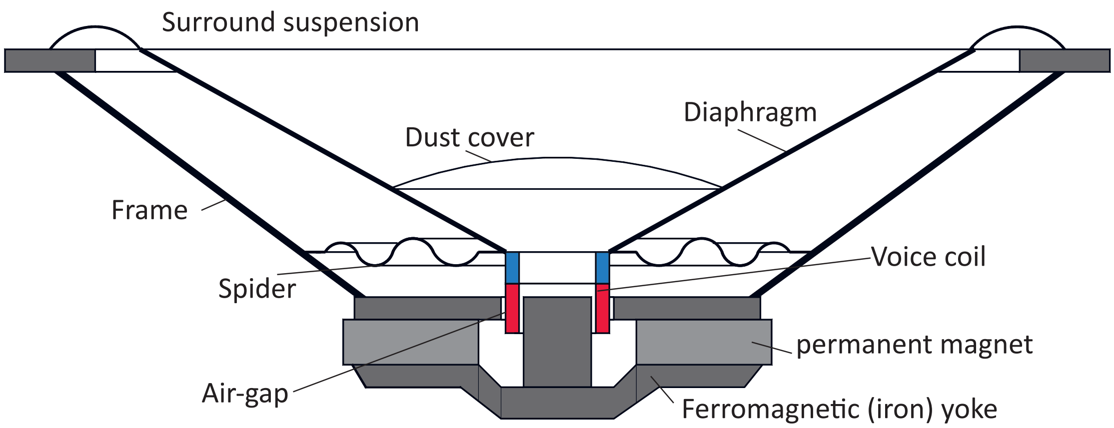

Figure 1: Cross section of an low frequency loudspeaker.

1 Introduction

A large number of sound generating transducers are developed over time, ranging from small vibrating membranes in a horn to the modulation of plasma by a varying magnetic field for high frequencies and even rotary subwoofers for extremely low frequencies. With the exception of the mentioned plasma principle most practical loudspeakers always apply an intermediate material, in most cases a flat or conical *diaphragm* that drives the air molecules based of forces from an actuator[1] The diaphragm can be rigid or non-rigid, depending on the means of actuation. The best example of the latter is the electrostatic loudspeaker where the thin non-rigid membrane is directly driven over its entire surface by electrostatic forces.
Most low frequency loudspeakers use a linear Lorentz-type actuator that drives the diaphragm, which is guided in one motion direction (degree of freedom) by an elastic suspension inside a supporting frame (Figure 1). The Lorentz-type actuator is also called a *moving-coil actuator* and consists of a permanent magnet "stator" creating a strong magnetic field inside an air-gap with a moving coil inside that air-gap that transforms the current in the coil into a force on the diaphragm which in its turn "pushes" the air.
Because of the fact that the electrodynamic loudspeaker was one of the first applications of Lorentz actuators, the moving coil is also called a *voice coil* because that coil gives the "voice" to the loudspeaker.

After a short comment on rotary subwoofers, the following sections first describe the physical relations that determine the radiated sound power for a vibrating plate, representative for the diaphragm of a low frequency loudspeaker. The second section describes the dynamics of amplifier, actuator and the mechanical system as they determine the vibrations of the diaphragm as function of the signal from the amplifier. Finally the impact of the enclosure is presented.

[1]Often the actuator of a loudspeaker is called a "motor". While both names are in principle correct, the name "actuator" is reserved for a system that exerts variable forces to a moving mass around a certain working point, while generally a "motor" relates to a driving system for a more continuous movement.

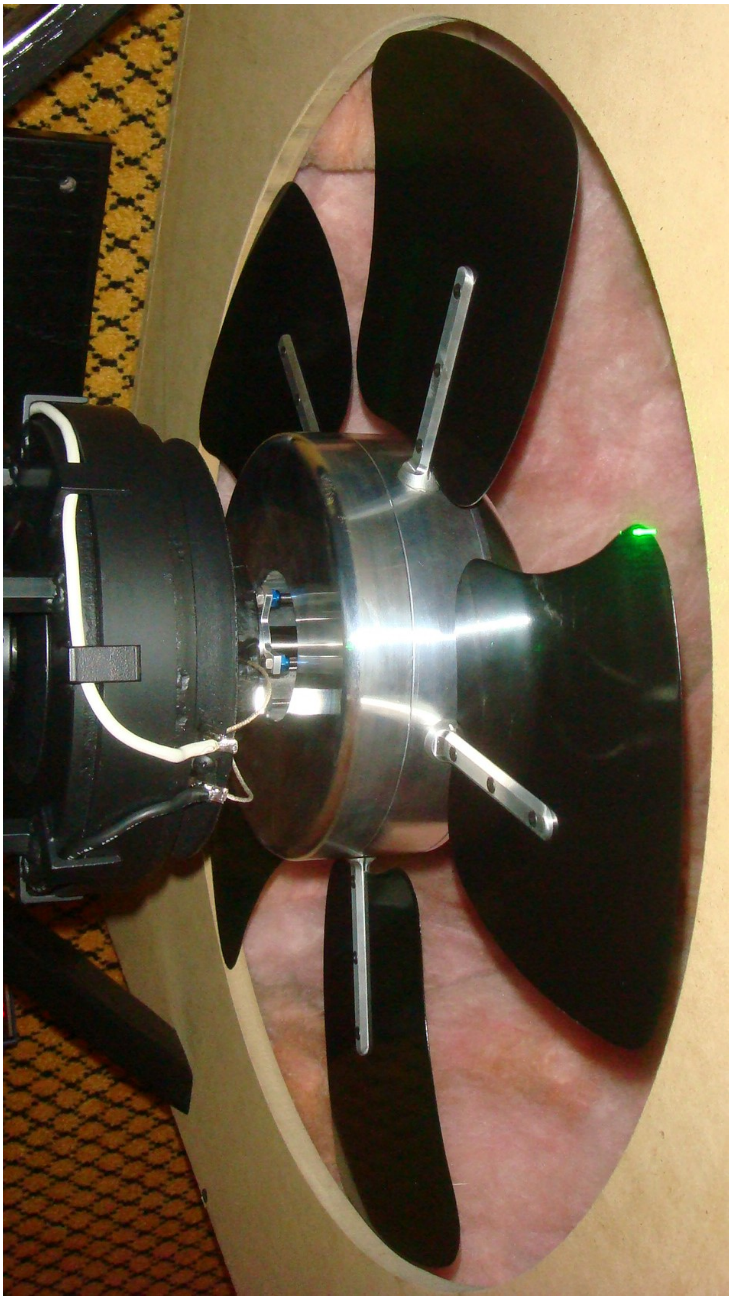

Figure 2: The Thigpen rotary subwoofer by Eminent Technology uses a fan with controllable pitch of the blades to create pressure variations between the front and the back side of the fan by changing the pitch of the blades. The back side faces a large enclosure volume, while the front side delivers the sound pressure via a second chamber covered with damping material to reduce the airflow noise.

1.1 Rotary Subwoofers

To understand physics phenomena in general it is often illustrative to observe extreme situations. With low frequency sound reproduction the static pressure at 0 Hz is such an extreme situation. It refers to a constant pressure that is lower or higher than the average environmental pressure. This phenomenon can only be obtained by pointing a continuous air flow towards an object, like wind in a sail. It becomes immediately clear from this extreme example that such a situation can

never be created by a diaphragm that moves over a limited range, as is the case in a "normal" loudspeaker driver.
It is however possible to generate an artificial wind by means of a fan as shown in Figure 2. By controlling the pitch of the blades the direction and the amount of air that flows through the device can be changed, theoretically to any frequency, even at 0 Hz. It follows the same principle as used in the propeller-drive of an airplane to reverse the thrust when braking. Unfortunately there are some caveats for this system of which the most important is the flow noise from the fan blades, which demands the use of a voluminous damping structure. In practice for acceptable noise levels such a subwoofer can only be applied with a large baffle board ("suskast" in Dutch), an anechoic enclosure with an opening to the listening room, internally covered with sufficient damping material.
For these reasons a rotary subwoofer is only applicable in professional installations, which allow large systems, like with cinema's or large electronic church organs. A good example of the latter is the Thigpen rotary subwoofers from Eminent technology, which are used in a real church with the OPUS 4 church organ of Marshall and Ogletree.
As this paper focuses on installations for music reproduction at home or in recording and mastering rooms in studios with frequencies ranging not lower than 16 Hz, the rotary subwoofer is not considered a viable option and in the following only the more regular driver configuration with a reciprocating diaphragm is presented.

2 Sound from a Vibrating Diaphragm

A loudspeaker driver diaphragm moves ideally like a piston, creating pressure waves that are propagated through the air.
To calculate the radiated sound power the loudspeaker is assumed to be mounted in such a way that the pressure from the back side can never reach the pressure from the front side. This can be achieved by means of an infinitely large plate, restricting the sound to be radiated in a hemisphere, by an infinitely large tube, where the sound will be radiated over a full sphere in space or by a closed chamber (enclosure) where the radiation varies from spherical at low frequencies to hemispherical at high frequencies.
The average sound power over one period, radiated from one side of the diaphragm moving with a sinusoidal motion, is equal to the multiplication of the effective (RMS) value[2] of the velocity of the diaphragm ($v_{\mathrm{d(rms)}} = \dot{x}_{\mathrm{d(rms)}}$) with the effective value of the real (in phase) part of the force on the diaphragm caused by the pressure that is exerted on the diaphragm.

$$P_{\mathrm{s}} = \dot{x}_{\mathrm{d}} F_{\mathrm{d(rms)}} = \dot{x}_{\mathrm{d(rms)}} p_{\mathrm{d(rms)}} S_{\mathrm{d}} \tag{1}$$

[2]The effective value is defined as the value of an equivalent DC value of the parameter that creates the same average power as the alternating version of the parameter. The effective value of a sinusoidal voltage equals $V_{\mathrm{eff}} = \hat{V}/\sqrt{2}$ with $\hat{V}$ = the amplitude of the voltage.

with:

P_s	=	sound power	[W]
$\dot{x}_{d,(rms)}$	=	effective value of the velocity of the diaphragm	[m/s]
$F_{d,(rms)}$	=	effective value of the force on the diaphragm	[N]
$p_{d,(rms)}$	=	pressure on diaphragm	[N/m^2]
S_d	=	surface of diaphragm	[m^2]

The pressure on the diaphragm is caused by the movement of the diaphragm itself working on the surrounding air with the following relation:

$$p_{d(rms)} = \dot{x}_{d(rms)} R_a \quad [\text{N/m}^2] \tag{2}$$

where the acoustic resistance R_a is equal to the real part of the complex, frequency dependent acoustic impedance[3] Z_a. Only the real part creates power as it corresponds with the component of the air pressure that is in phase with the velocity[4].
The acoustic resistance is frequency dependent at low frequencies and becomes constant at higher frequencies. From empirical analysis it is found that the values can be approximated as follows:

$$R_{a,LF} \approx \frac{\rho_0 S_d \omega^2}{2\pi c_0} \quad [\text{Ns/m}^5] \tag{3}$$

$$R_{a,HF} \approx \frac{\rho_0 S_d c_0}{\pi d_d^2} \quad [\text{Ns/m}^5] \tag{4}$$

with:

ρ_0	=	density of air	[kg/m^3]
ω	=	angular frequency	[rad/s]
c_0	=	propagation velocity of sound waves	[m/s]
d_d^2	=	diameter of the diaphragm	[m]

The transition frequency f_t between the low and high frequency range is found when both values are equal:

$$f_t = \frac{\omega_t}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2\pi c_0^2 \rho_0 S_d}{\pi d_d^2 \rho_0 S_d}} = \frac{c_0}{\pi d_d \sqrt{2}} \quad [\text{Hz}] \tag{5}$$

A large loudspeaker with a diaphragm diameter of 0.3 m with the velocity of sound $c_0 = 340$ m/s would have a transition frequency of ≈ 250 Hz, which indicates that for

[3] In some other literature the impedance is defined as the ratio between the force over the diaphragm instead of the pressure on the diaphragm versus the velocity. In that case S_d appears squared in the acoustic resistance and not in Equation(6). The resulting expressions for the force and the radiated average sound power are then of course identical.

[4] The real sound velocity and pressure have a complex relation and are both proportional to the square root of the sound power. Even though the pressure and velocity of the diaphragm have a direct relation with the produced sound pressure at some distance, they are not identical.

low frequency sound reproduction with subwoofers it is allowed to use the value from Equation (3).

With these values and equations the force by the air on the diaphragm can be calculated:

$$F_{\mathrm{d,(rms)}} = p_{\mathrm{d,(rms)}} S_{\mathrm{d}} = \dot{x}_{\mathrm{d,(rms)}} S_{\mathrm{d}} R_{\mathrm{a}} \ \ [\mathrm{N}] \tag{6}$$

Using Equation (3) this is equal to:

$$F_{\mathrm{d,(rms)}} = \dot{x}_{\mathrm{d,(rms)}} \omega^2 \frac{S_{\mathrm{d}}^2 \rho_0}{2\pi c_0} \ \ [\mathrm{N}] \tag{7}$$

The average radiated sound power is equal to:

$$P_{\mathrm{s}} = \dot{x}_{\mathrm{d,(rms)}} F_{\mathrm{d,(rms)}} = \dot{x}_{\mathrm{d,(rms)}}^2 \omega^2 \frac{S_{\mathrm{d}}^2 \rho_0}{2\pi c_0} \ \ [\mathrm{W}] \tag{8}$$

With a sinusoidal reciprocating motion of the diaphragm, $v_{\mathrm{d,(rms)}} = x_{\mathrm{d,(rms)}} \omega$ and the average sound power can be written as:

$$P_{\mathrm{s}} = x_{\mathrm{d,(rms)}}^2 \omega^4 \frac{S_{\mathrm{d}}^2 \rho_0}{2\pi c_0} \ \ [\mathrm{W}] \tag{9}$$

When the diaphragm moves with a constant amplitude for all frequencies, the radiated power increases proportional with frequency to the power 4. In control-engineering terms this represents a slope of +2 in the frequency response plot being +40 dB/decade.
A frequency independent output power level would require that the displacement amplitude is inversely proportional to the frequency squared. This requirement has the following important consequences:

- A high output power level at low frequencies requires a large displacement amplitude.
- The displacement amplitude can only be reduced by a larger surface of the diaphragm.

This is the reason why powerful low frequency loudspeakers need to be large.

Another important aspect is the relation between the sound power and acceleration. Even though the sound power is not generated by the acceleration, the squared relation to the amplitude of the displacement, as found in Equation (9), means that the radiated power is proportional to the acceleration of the diaphragm as also the acceleration increases with the frequency squared with a slope of 40 dB/decade when the amplitude of the displacement is kept constant.
With the effective value of the acceleration $\ddot{x}_{\mathrm{d,(rms)}} = x_{\mathrm{d,(rms)}} \omega^2$, Equation (9) can be written as the following frequency independent relation:

$$P_{\mathrm{s}} = \ddot{x}_{\mathrm{d,(rms)}}^2 \frac{S_{\mathrm{d}}^2 \rho_0}{2\pi c_0} \ \ [\mathrm{W}] \tag{10}$$

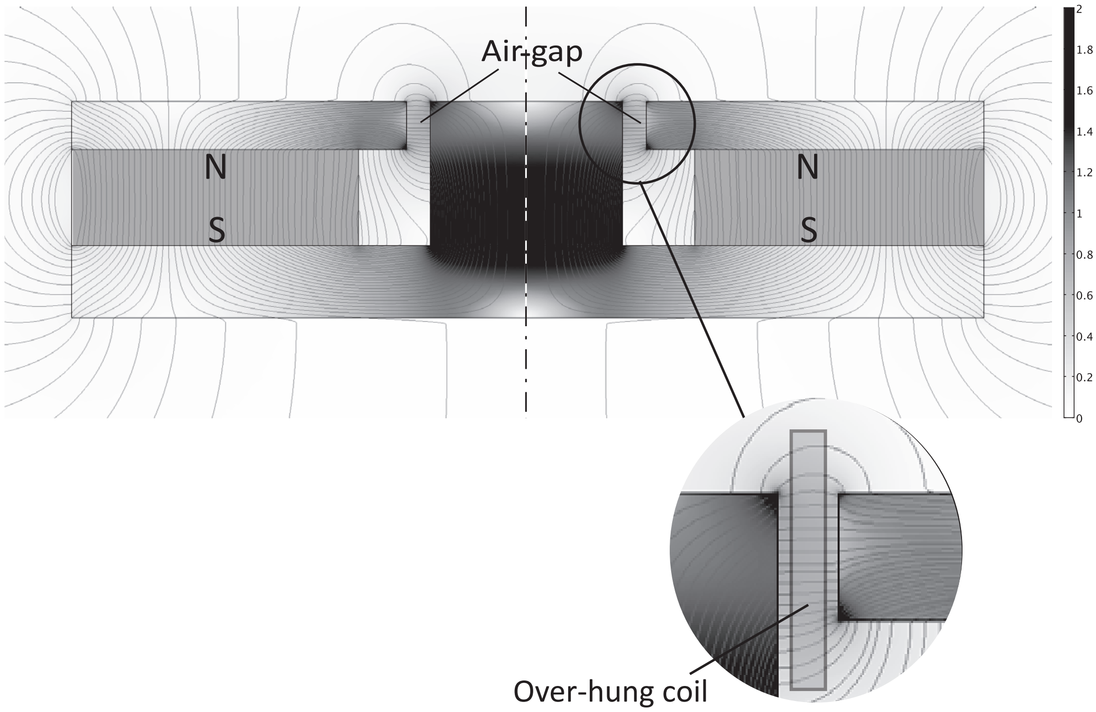

Figure 3: Cross section of a voice coil actuator consisting of a permanent magnetic structure, which generates a strong magnetic field in a circular air-gap, and a moving coil, which is inserted in this air-gap. A larger (overhung) coil than the air-gap increases the range over which the force is more constant.

This means that a diaphragm will produce a constant frequency independent sound power when the acceleration is kept at a constant level.
Furthermore the sound pressure is proportional to the square root of the sound power which means:

$$p_{\mathrm{s}} \propto \sqrt{P_{\mathrm{s}}} \propto \ddot{x}_{\mathrm{d,(rms)}} \tag{11}$$

This relation of sound pressure with acceleration at low frequencies is an important phenomenon as it means that the dynamic behaviour of a loudspeaker can be mastered by controlling the acceleration of the diaphragm.

3 The Lorentz-type actuator

The Dutch physicist and Nobel prize winner Hendrik Antoon Lorentz (1853 – 1928) formulated the Lorentz force as a completion to the Maxwell equations. The law of Faraday describes the effect of a changing magnetic field on electrical charges hence generating electricity from kinetic energy. Based on energy conservation laws creating electrical energy from motion is fully complementary to creating motion energy from electrical energy so the laws of Lorentz and Faraday are strongly related. In vectorial notation the formulation of Lorentz describes the force on a moving charged particle as:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{12}$$

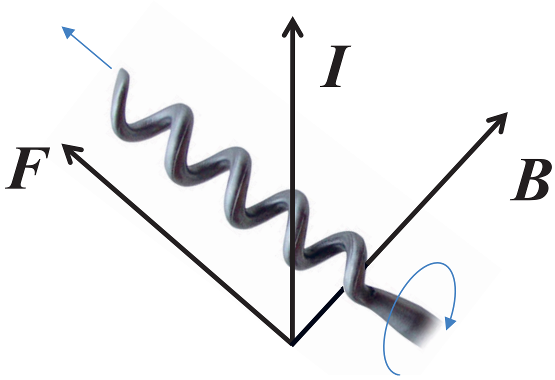

Figure 4: Determining the direction of the Lorentz force with the corkscrew rule. When the corkscrew is rotated right handed, from the direction of the positive current to the direction of the magnetic field (arrow), the movement of the point of the corkscrew determines the direction of the force.

with v [m/s] equals the instantaneous velocity of the particle. The first part of the Equation $q\boldsymbol{E}$ is the electrostatic force and the second part is the electromagnetic force. This second term is used in electromagnetic actuators. Next to the force on a moving particle it equally represents the force on a current flowing through a wire with length ℓ_w [m], inserted in the magnetic field. For this situation the moving charge equals the current times the length, $qv = \ell_\mathrm{w}\boldsymbol{I}$, and with this relation the electromagnetic Lorentz force is equal to:

$$\boldsymbol{F} = \ell_w \boldsymbol{I} \times \boldsymbol{B} \tag{13}$$

For the magnetic force on a wire at an angle α relative to the direction of a magnetic field with flux density B, carrying a current I, this relation leads to the scalar notation of the Lorentz force of electromagnetic actuators of which the magnitude is given by:

$$F = BI\ell_\mathrm{w}\sin\alpha \tag{14}$$

The direction of this force is orthogonal to the plane that is determined by the direction of the magnetic field and the current, due to the “cross product” in the vectorial Lorentz equation. This rule can be remembered as the *right hand* or *corkscrew* rule, which states that the positive force direction is found when rotating a corkscrew from the positive current direction onto the direction of the magnetic field as shown in Figure 4. Of course for a real mechanical engineer any normal right turning screw will also suit the purpose, but the corkscrew is more easy to remember.

In most practical cases the Lorentz force must be maximised, which means that $\sin\alpha$ is kept as much as possible equal to one. This means that the simplified equation becomes equal to:

$$F = BI\ell_\mathrm{w} \tag{15}$$

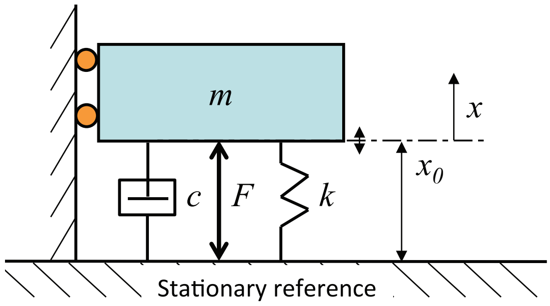

Figure 5: A damped mass-spring system with an external force stimulus.

And with multiple windings the Lorentz force becomes:

$$F = BIn\ell_{\mathrm{w}} = BI\ell_{\mathrm{w,t}} \tag{16}$$

where $\ell_{\mathrm{w,t}}$ equals the total length of the wire inserted in the magnetic field. When divided by the current I this equation gives the force to current ratio, also called the *force constant*, of the actuator, which will prove to be not constant at all (see paper "Distortion Sources in Loudspeaker Drivers".

4 Dynamic Properties of Loudspeaker Drivers

For low frequencies a moving coil loudspeaker driver can be described in a simple model as shown in Figure 5. The moving body with mass m consists of the diaphragm and the coil. The body is suspended by the spider and the rubber roll surround, with a certain stiffness k. Finally the damper, with damping coefficient c consists of the sound radiation, the rubber surround and the damping caused by actuator-amplifier combination. With Equation (31) the damping due to the sound can be calculated. It is a frequency dependent value so it is necessary to calculate it for a certain frequency. As will become clear in the actual example however, the acoustic damping is so low in respect to the other causes of damping that it is allowed to neglect the acoustic effects on mass and damping. Also the effect of the surround is small compared with the electromagnetic damping of the actuator in combination with the amplifier.
To determine the dynamic behaviour of the total system, the electrical circuit of Figure 6 is used. For the electrical signal source to the loudspeaker a usually applied amplifier with a voltage source output is chosen. A voltage source output means that the amplifier has a very low output impedance $\mathrm{R_s}$ approximating 0 Ω, (in practice < 100 mΩ).
The electrical circuit of the loudspeaker can be approximated as a series circuit of the resistance and self inductance of the coil windings with a motion voltage source, which is proportional to the velocity of the coil relative to the magnetic field.
When connecting the loudspeaker to a voltage amplifier, the voltage source of the amplifier becomes connected in series with the motion voltage source and the total

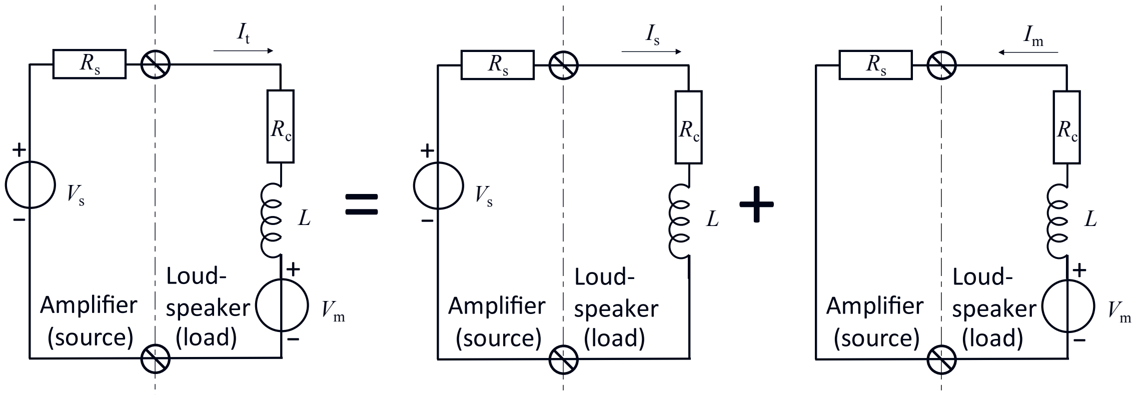

Figure 6: The electrical model of the amplifier loudspeaker combination. The total current (I_t) though the loudspeaker consists of the part (I_s) delivered by the voltage source of the amplifier and the part (I_m) delivered by the motion induced voltage of the loudspeaker coil.

impedance. This circuit determines the current (I_t) that creates the force to the moving part of the loudspeaker. The effect of both voltages on the total current can be analysed separately as their current contributions to the circuit can be super imposed because of the linear properties of the circuit in this approximation. In practice the frequency area where the effect on damping is large is so low that the self inductance can be neglected for the analysis and only the resistive element with value $R = R_s + R_c \approx R_c$ is considered.

4.1 Amplifier Voltage Response

First the effect of the amplifier voltage is determined only by replacing the motor voltage by a short circuit which is allowed as the impedance of a voltage source is zero. The force of the motor is found by using Ohm's law and the force relation of a Lorentz actuator.

$$F = BI_s\ell = \frac{BV_s\ell}{R_c} \quad \text{[N]} \tag{17}$$

where B equals the magnetic flux density in Tesla [T] through the coil and ℓ equals the length of the windings of the coil inside the magnetic field. From dynamic analysis the frequency response $T_{F,x}$ of the cone excursion x with mass m to an excitation force F is given as follows as function of radial frequency $\omega = 2\pi f$:

$$T_{F,x}(\omega) = \frac{x}{F} = \frac{C_s}{-\frac{\omega^2}{\omega_0^2} + 2j\zeta\frac{\omega}{\omega_0} + 1} \tag{18}$$

The defined damping ratio ζ, compliance C_s and resonating eigenfrequency[5] ω_0 are equal to:

$$\zeta = \frac{c}{2\sqrt{km}} \quad C_\mathrm{s} = \frac{1}{k} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

The resonating eigenfrequency f_0 in hertz is equal to $\omega_0/2\pi$ and is called by different names like the first or fundamental resonance frequency, because at higher frequencies many additional resonances occur in a loudspeaker. At frequencies below $f_0 = \omega_0/2\pi$, the first two terms in the denominator of Equation (18) will become small relative to one and the frequency response approaches the constant factor C_s. This means that the magnitude of the cone excursion becomes frequency independent for a given excitation force. This is the dynamic situation where the force of the loudspeaker actuator is in balance with the force due to the motion of the cone against the stiffness of the loudspeaker suspension combined with the air stiffness of the enclosure.
At frequencies above $f_0 = \omega_0/2\pi$ the first term in the denominator of Equation (18) becomes dominant and the magnitude of the cone excursion becomes inverse proportional to the frequency squared. This is the dynamic situation where the force of the loudspeaker actuator is in balance with the acceleration of the mass of the cone.
At $f_0 = \omega_0/2\pi$ the magnitude of the cone excursion can become very large, in theory even infinite if $\zeta = 0$. This is the low-end resonance frequency of any electrodynamic loudspeaker.

The acoustic response of a loudspeaker is shown to be proportional to acceleration which means that the frequency response of Equation (18) has to be combined with the corresponding +2 (40dB/decade) slope in the frequency response that is related to acceleration as being the second derivative of the position. This combination is obtained by multiplication of Equation (18) with ω^2. Together with Equation 17 the frequency response $T_{\mathrm{V},\ddot{x}}$ from voltage to acceleration becomes as function of radial frequency $\omega = 2\pi f$:

$$T_{\mathrm{V},\ddot{x}} = -\frac{\omega^2 x}{V_\mathrm{s}} = -\frac{\dfrac{\omega^2 C_\mathrm{s} B\ell}{R_\mathrm{c}}}{-\dfrac{\omega^2}{\omega_0^2} + 2j\zeta\dfrac{\omega}{\omega_0} + 1} = -\frac{\omega^2 K_\mathrm{e}}{-\dfrac{\omega^2}{\omega_0^2} + 2j\zeta\dfrac{\omega}{\omega_0} + 1} \tag{19}$$

With:

$$K_\mathrm{e} = \frac{C_\mathrm{s} B\ell}{R_\mathrm{c}} \tag{20}$$

Figure 7 shows this relation in an amplitude and phase to frequency response normalised to $K_\mathrm{e} = 1$. It is clear that for frequencies below $f_0 = \omega_0/2\pi$ the original flat

[5] A resonance frequency of a mechanical structure is called "eigenfrequency" because it is an intrinsic (Dutch and German "eigen" means own) dynamic property of the structure.

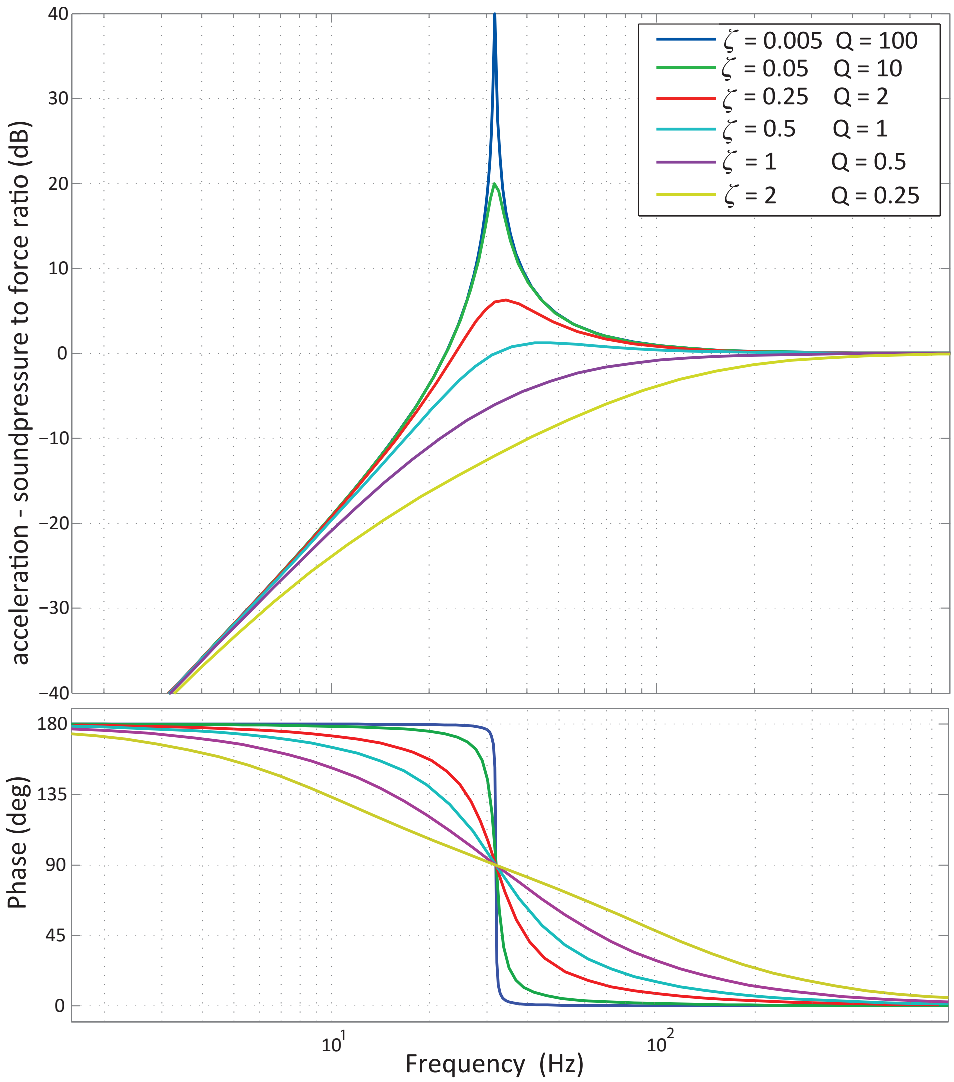

Figure 7: Frequency response of the radiated sound as function of a periodic excitation force of an electrodynamic loudspeaker, normalised to $K_e = 1$ with different damping settings.

response now has become a low frequency roll-off with a slope of +2 and 180° phase lead, while the response above $f_0 = \omega_0/2\pi$ has become frequency independent without a phase lead.

At ω_0 the damping determines the response and in this graph both the damping ratio ζ and the quality factor Q are mentioned as Q is mostly used in loudspeaker systems. These terms relate as follows:

$$Q = \frac{1}{2\zeta} = \frac{\sqrt{km}}{c} \tag{21}$$

4.2 Motion Voltage Response

The next step is to determine the force by the current (I_m) that is induced by the motion voltage of the moving coil through the amplifier that in its turn can be approximated as a low impedance circuit. This current value is determined by the motion voltage and R_c and causes a force that counteracts the movement so it acts like a damper.
The motion induced voltage of a moving coil with velocity v in a magnetic field equals:

$$V_m = B\ell v \text{ [V]} \tag{22}$$

The resulting current equals according to Ohm's law:

$$I_m = \frac{V_m}{R_c} = \frac{B\ell v}{R_c} \text{ [A]} \tag{23}$$

As the current will flow in the same magnetic field, a damping Lorentz force will occur:

$$F_d = B\ell I_m \text{ [N]} \tag{24}$$

With the previously derived value for I_m this gives:

$$F_d = \frac{(B\ell)^2 v}{R_c} \text{ [N]} \tag{25}$$

So the damping coefficient c is:

$$c = \frac{F_d}{v} = \frac{(B\ell)^2}{R_c} \text{ [Ns/m]} \tag{26}$$

Combining this damping coefficient with the spring-stiffness and mass gives the electrical Q factor given in the data sheets.

$$Q = \frac{\sqrt{km}}{c} = \frac{\sqrt{kmR_c}}{(B\ell)^2} \tag{27}$$

It is good to notice that the damping effect is in fact identical to velocity feedback as it is a force that proportionally counteracts velocity. This is further explained in the paper "Motional Feedback Theory in a Nutshell".

5 Using an Enclosure

The equations of the previous section on acoustic radiation apply to the situation where all energy is radiated from the front and not from the back side. The sound pressure from one side of the diaphragm is radiated in counter phase to the sound pressure from the other side because a rise in pressure at the front surface will correspond with a sink in pressure on the backside and the other way around. At low frequencies the pressures have ample time to reach the opposite side when propagating at the velocity of sound. As a result destructive interference occurs between both pressures, which means that the resulting pressure difference is reduced by cancellation. This phenomenon is called *acoustic short-circuiting* and at 0 Hz the cancellation is complete. At higher frequencies the cancellation will gradually reduce giving a +1 slope in the frequency response until the frequency where the distance d from front to back equals half a wavelength ($d = 0.5 \cdot 340/f_1$), giving 180° phase shift with constructive interference for that frequency. At $f_2 = 2f_1$ so double the frequency the distance between front and backside equals a full wavelength with 360° phase and destructive interference again. With higher frequencies this succession of constructive and destructive interference will happen at equal frequency intervals which on the logarithmic scale of a frequency response plot will appear as "hair" at higher frequencies. This all is shown in Figure 8 and the characteristic frequency response is called a *Comb filter* as it would show up in a non-logarithmic frequency plot as equally spaced spikes. This basic reasoning neglects the fact that the sound is created over the entire diaphragm surface and not only at the centre but anyway the sound radiation becomes irregular and the low frequency radiation is strongly reduced. In fact the radiation pattern of an unmounted loudspeaker will always look like a dipole. On-axis the sound pressure is maximum, while off-axis at both sides when for instance $\lambda = 2d$ and on the intermediate radiating plane the sound pressure is zero for all frequencies. This observation matches the situation when measuring the frequency response in an anechoic room without reflections but in a real living room the walls will reflect the sound and in practice some loudspeaker systems are designed such, that they use these reflections to be able to compensate the loss of sound pressure due to acoustic interference. These so called "open baffle" systems have often a very appreciated sound although the placement in the room is even more critical than when the loudspeaker is built in an enclosure. To avoid these problems, generally the loudspeaker is mounted in an enclosure, which cancels the sound from the back by blocking (closed-box), dissipates the sound over a long path, which mimics an infinite enclosure (transmission line) or uses an additional resonator (bass-reflex), which reduces the movement of the diaphragm at its resonance frequency and filters the radiation from the back side. All these methods have their benefits and drawbacks. The transmission line requires a very large volume due to the need to fit at least one half wavelength of the lowest frequency (7,5 m for 20 Hz) because always an anti-node (high velocity) of the sound must be captured in the damping material, which is very impractical. The closed box has a well controlled

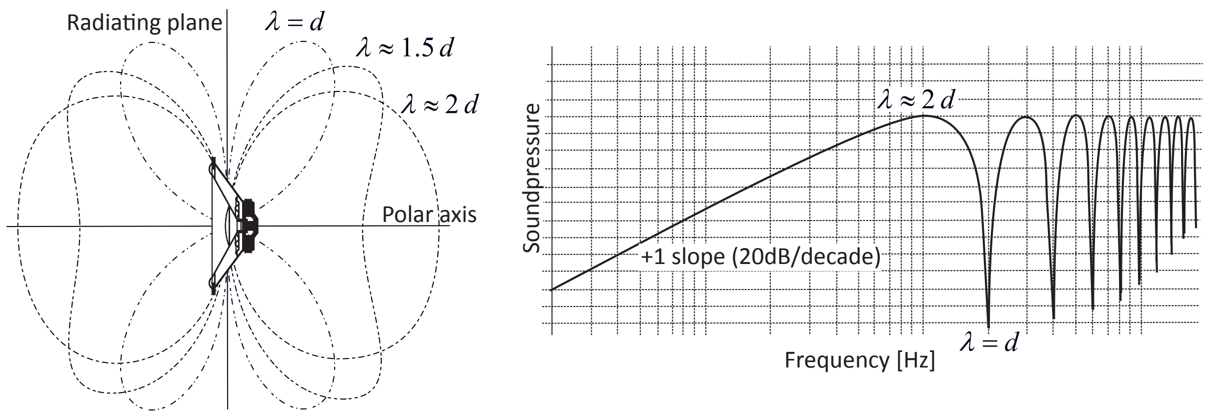

Figure 8: An electrodynamic loudspeaker radiates sound in two directions with 180° difference. Without an enclosure that stops one of the sound waves they will interfere with each other causing a frequency related spatial dipole radiation pattern and an irregular frequency response on axis, called a "comb filter". The graphs do not include the dynamics of the loudspeaker so a constant acceleration amplitude is assumed over the shown frequency band.

behaviour but the stiffness of the air spring by the enclosed volume will increase the fundamental resonance frequency with a corresponding decrease of damping (= increase of Q factor). The bassreflex system allows more acoustic power at the lowest bandwidth frequency, but it introduces a delay in the response due to the resonance and a 4^{th}-order roll off below the lowest badwidth frequency. The bass reflex principle is presented in a separate paper and the conclusion is that it is not useful for low frequency subwoofers. While also the transmission line is not useful due to its size, only the closed-box enclosure is further treated in this paper.

5.1 Closed-Box Enclosure

When a loudspeaker is mounted in a closed-box enclosure the air inside the box can not escape nor can the sound pressure from the back side of the diaphragm reach the front side. A drawback of the closed-box configuration is the fact that the sound from the back side of the diaphragm can also not reach our ears. For high frequencies, where the size of the enclosure is large in respect to the wavelength, this sound from the back side should be absorbed by damping material as otherwise that sound would be reflected by the inner wall of the enclosure and return through the diaphragm, which is very light. This mixing of direct sound from the front side of the diaphragm and reflected sound from the back side in an enclosure causes a diffuse sound. For this reason (and to reduce the stiffness by isothermal expansion as shown in the following section) a piece of damping material is applied inside the enclosure to absorb the radiated energy. Fortunately, at low frequencies with long wavelengths larger than the size of the enclosure, this is not a problem. This can be understood from the following reasoning, while using the relations given in Section 2. In that section the radiated sound power of a diaphragm in free air was

derived from the velocity of the diaphragm and the pressure on the diaphragm. This pressure appeared to be related to the velocity, thereby creating power, because the velocity and the pressure are in phase.
Inside an enclosure with small dimensions relative to the wavelength, the pressure is only related to the position of the diaphragm as the enclosed air acts as a spring. With sinusoidal movements the velocity is 90 degrees out of phase with the position, because when the position $x(t) = A \sin\omega(t)$, the velocity equals $v(t) = \mathrm{d}x/\mathrm{d}t = A\omega\cos\omega(t)$ and cos differs 90 degrees from sine. The multiplication of cos and sine of the same term, which is the case when determining the power, averages to zero as follows from the following trigonometric identity: $\sin\omega(t)\cos\omega(t) = \sin(2\omega(t))/2$, which is average zero. As a consequence the acoustic power at the back side is zero for low frequencies.

5.1.1 Impact of Stiffness of the enclosed Air

The enclosed air in the box will act as an air spring that adds its stiffness to the stiffness of the suspension of the loudspeaker diaphragm resulting in a higher fundamental resonance frequency, thereby limiting the low frequency bandwidth. For that reason it is important to determine this stiffness. To calculate this coupling stiffness by the air in the enclosure, the pressure on the surface area of the diaphragm S_d of a loudspeaker is calculated as function of the displacement. The compression/expansion is somewhere between adiabatic and isothermal. When the cabinet is filled with fibre wadding like wool or any other material consisting of soft fine fibre like polyester, the temperature will fluctuate less at expansion/compresssion. In that case the relation between the pressure and the compressed volume can be derived by applying the simple isothermal gas law $PV = C$. For adiabatic expansion/compression the pressure change is larger due to the temperature rise at compression. For air this is a factor $\lambda = 1.4$. This factor will linearly increase the stiffness of the air spring with a resulting higher fundamental resonance frequency of the combination of the moving mass of the diaphragm with this stiffness. This resonance frequency should be as low as possible for an extended low frequency bandwidth of the loudspeaker. A slight reduction of the stiffness can be achieved by making the expansion/compression more isothermal by filling the cabinet with very fine fibre padding, even when this padding is not required for other reasons like the abovementioned damping of internal reflections and standing waves inside the cabinet at higher frequencies. In practice a reduction of λ to approximately 1.2 can be achieved when applying padding fibre giving a reduction of the resonance frequency with approximately 10%.
Suppose δx displacement of the diaphragm, the displaced volume δV equals:

$$\delta V = S_\mathrm{d}\delta x \quad [\mathrm{m}^3] \tag{28}$$

The displaced volume will cause a relative pressure change to the environmental pressure P_0 that can be calculated with the adiabatic gas law and the relative change of the volume of the enclosure V_e:

$$\frac{\delta P}{P_0} = \lambda \frac{\delta V}{V_\mathrm{e}} \implies \delta P = \lambda \delta V \frac{P_0}{V_\mathrm{e}} = \lambda S_\mathrm{d} \delta x \frac{P_0}{V_\mathrm{e}} \quad [\mathrm{N/m^2}] \tag{29}$$

With the surface area of the diaphragm S_d, the stiffness of the spring due to the enclosed air is equal to the following expression:

$$\begin{aligned} k_\mathrm{a} &= \frac{F_\mathrm{d}}{\delta x} = \frac{\delta P S_\mathrm{d}}{\delta x} = \lambda S_\mathrm{d} \delta x \frac{P_0}{V_\mathrm{e}} \frac{S_\mathrm{d}}{\delta x} \\ &= \lambda S_\mathrm{d}^2 \frac{P_0}{V_\mathrm{e}} \quad [\mathrm{N/m}] \end{aligned} \tag{30}$$

By using the calculated stiffness value in the relations that were presented in Section 4 it can be concluded that the resonance frequency of a loudspeaker will increase when mounted in a closed-box enclosure, while the damping ratio ζ will be decreased, giving a larger Q-value. In practical designs mostly the fundamental resonance frequency is increased with at least a factor two, which is significant.

To improve this behaviour there are several possibilities:

- Compensate the characteristics with an electronic filter with an inverse transfer function.
- Measure the sound and use acoustical feedback.
- Measure the motion of the diaphragm and use “motional feedback”.

The use of an **inverse filter** is the mostly used principle although it is hardly possible to do this accurately especially with situations where the damping is even lower. The exact tuning of the dynamic response including overshoot is for that reason often less than acceptable. The use of digital electronics has made it possible to realise a more exact inverse transfer function by means of Digital Signal Processors (DSPs) and built in amplifiers dedicated for each loudspeaker. In the first example that will be shown a sophisticated DSP compensator reaches the best one could obtain with this method. A remaining problem with this approach is that it is purely based on feed-forward compensation while it will force the loudspeaker to very high uncontrolled excursions at very low frequencies into the non-linear range of the surround spring and the Lorentz actuator. This will cause severe distortion unless a very good (and thus expensive) loudspeaker is used. Still the related distortion is quite often accepted as people tend to like distortion. This is caused by a special property of human ears that distort themselves quite heavily as function of the loudness. A certain level of distortion will in reality be perceived as a louder signal while also the human brain can reconstruct the fundamental frequency from its harmonics giving the perception of a lower signal frequency even if this lower frequency is reduced in amplitude. This is why a small transistor radio still gives the right impression of the total music as otherwise the lower tones would be perceived as one or two octaves higher changing a baritone in a soprano. Yet it is not preferred as the bass is missing and the distortion and resonances will cause headache.

The second method is to **measure the sound** itself and use that as feedback signal for an actively controlled system. This approach is limited by the delay between the loudspeaker and the microphone and the room reflections that are also delayed and detected by the microphone. Although this principle can be used with adequate filters to suppress room reverberations, which often deteriorate the sound quality, these delays would cause the system to become unstable resulting in a howling sound. Furthermore it would be limited to the very low frequencies where the wavelength is long due to the delays and lack of coherence (phase relationship) in the reflected sound waves.

The best method is to use active feedback control of the moving diaphragm and is in general terms presented in a separate paper on "Motional Feedback in a Nutshell". All active feedback controlled systems are based on the closed-box enclosure as it behaves the most deterministic.

5.1.2 Efficiency

Loudspeakers are normally designed to be operated with a voltage controlled amplifier with a near zero output impedance. Often more loudspeakers are mounted together in one enclosure to cover different frequency ranges which means that these loudspeakers have to match in sound intensity (power). In order to achieve this matching the radiated sound level of a loudspeaker is measured with a standard signal. This standard signal is the AC voltage level that would result in an electrical Power of 1 Watt in a 8 Ohm pure resistance. The corresponding voltage has an effective value of 2.83 V_{rms}.

To get a feel of the efficiency of subwoofers some average numbers will illustrate the fact that an electrodynamic loudspeaker has an extremely low efficiency in converting electrical power into acoustical power. Most subwoofers have a coil resistance in the order of 4 Ohms, which means that the standard reference voltage of 2.83 V_{rms} represents an input power level of 2 W. The specified acoustic power of many loudspeaker @1 metre over a hemisphere ranges in practice between 80 and 90 dB for this reference input voltage. The best case value of 90 dB is 30 dB below 120 dB, which represents a sound power intensity of $1W/m^2$. A -30 dB lower power level is equivalent to 10^{-3}, which means that 90 dB is equal to a sound power intensity of 1 mW/m^2. With a surface of a hemisphere @ 1 m of 6.3 m^2 the total radiated power becomes 6.3 mW. With 2 W input power this is an efficiency of only 0.3%.

Another way to look at this low efficiency is in the damping effect on the fundamental resonance by the radiated sound. A high level of damping would imply a high level of radiated energy.

This acoustic damping can be derived from Equation (7):

$$c_{\mathrm{a}} = \frac{F_{\mathrm{d,(rms)}}}{\dot{x}_{\mathrm{d,(rms)}}} = \omega^2 \frac{S_{\mathrm{d}}^2 \rho_0}{2\pi c_0} \quad [\mathrm{Ns/m}] \tag{31}$$

By the following example with some practical values it will become clear that this acoustic damping is very limited, which also indicates the low efficiency of an electrodynamic loudspeaker.

S_{d}	=	$500 * 10^{-4}$	[m^2]
f	=	40	[Hz]
ω	=	$2\pi f \approx 250$	[rad/s]
ρ_0	=	1,2	[$\mathrm{kg/m}^3$]
c_0	=	340	[m/s]

This results in:

$$c_{\mathrm{a}} = \frac{(5 \cdot 10^{-2})^2 \cdot 1.2 \cdot 250^2}{2\pi \cdot 340} = 0.09 \quad [\mathrm{Ns/m}] \tag{32}$$

This value is obviously so small in respect to the electromagnetic damping with a voltage source amplifier with practical values of of 10-100 Ns/m that it can be neglected and this further underlines the weak spot in the bad energy efficiency of an electrodynamic loudspeaker.

Fortunately the sound level in real music has a very high dynamic range, where the average power is often at least 20 dB below the loudest peak level. This difference represents a factor 100 in power so even with a 1000 W amplifier, reducing the average 10 W by increasing the efficiency of the loudspeaker will not solve the energy problem of the earth. Nevertheless it is useful to get some understanding of the underlying reasons for this low efficiency.

5.1.3 Causes of the Low Efficiency

The first factor of the low efficiency is due to the need for low distortion. As described in another paper on distortion a longer actuator coil than the air gap is chosen in the design of subwoofers in order to keep the force of the actuator more constant over its movement range. The main cause, however, of the low efficiency of a loudspeaker driver is in the bad transfer of energy from the actuator to the moving diaphragm and the bad coupling from the diaphragm to the air. Starting with the relatively high mass of the diaphragm itself including the actuator coil when compared to the moved air, it is easy to imagine the bad coupling of the diaphragm to the air.
The transfer of the power P from the actuator to the membrane is a bit more difficult. First of all the transferred power is equal to the driving force F multiplied by the velocity v: $P = Fv$.
In Figure 9 the velocity of the membrane is shown as function of the driving force.

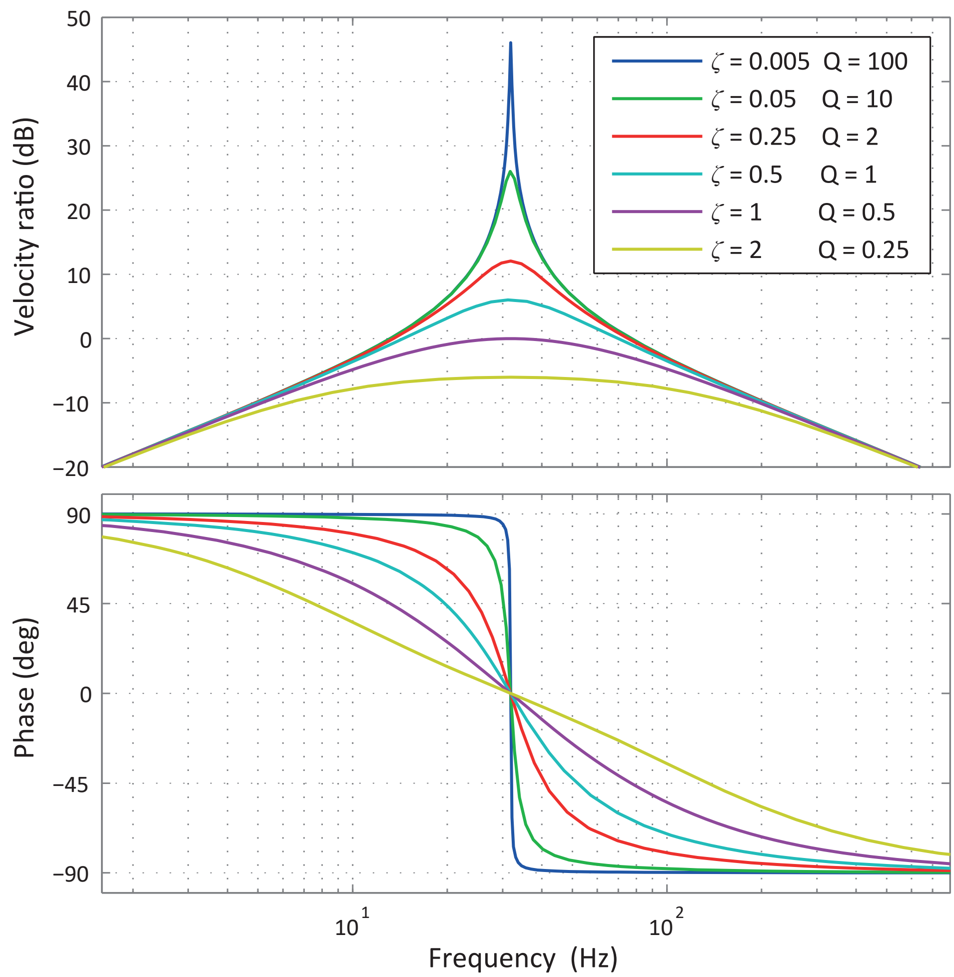

Figure 9: Frequency response of the diaphragm velocity as function of a periodic excitation force of an electrodynamic loudspeaker, normalised to one at higher frequencies with different damping settings. The phase relation shows that the velocity is more in phase with the force at elevated damping levels, indicated the higher energy dissipation by the damping.

This figure is made with the same driver parameters as Figure 7. It shows that at resonance the velocity is maximum matching a high energy transfer, which is the physical reason why there is a resonance anyway. It also shows that the energy transfer decreases with increasing damping, which is also logical as damping extracts energy and dissipates it into heat. It was shown in the previous section that this damping is hardly caused by the radiated sound of the air so it is only energy that is dissipated in the actuator coil.

This all makes clear that the efficiency can only be improved by increasing the extraction of energy by acoustic power.

5.1.4 Horn Shaped Impedance Transformer

In those cases, where it is required to limit the dissipated heat and electrical power for extreme levels of sound power, a well known method to improve the acoustic

Figure 10: The extreme consequences of a bass horn in a home environment. (source: `http://vincent.brient.free.fr/bass_horn.htm`)

coupling to the air is to use a horn. A horn acts as a lossless acoustical impedance transformer that converts the power of a high pressure with a low velocity level in the throat, the narrow part of the horn near the diaphragm, into a combination of normal sound related higher velocity and lower pressure at the wide part. The high pressure in the throat represents a larger acoustic impedance at the diaphragm with a resulting higher output power from its motion. In a musical instrument like a trumpet this enables the musician to create a strong sound with limited vibrations of his or hers lips, while in a loudspeaker the high pressure combined with the velocity of the diaphragm increases the efficiency. Loudspeakers with a horn load are well known from the sound systems at outdoor events, also known as "public address" and the large speaker systems of stage musicians. In these applications also the coil-to-air-gap length-ratio of the Lorentz actuator is chosen more closely matched (less or no overhung), which enhances the $B\ell$-factor, while sacrificing linearity. The related distortion is not regarded as a problem in these cases and is even welcomed in the music scene as it introduces higher harmonics which enriches the sound. One problem when using a horn is that the size should be proportional to the maximum wavelength which limits the use at low frequencies (15 metre for 20 Hz). This can be partly solved by folding the horn like in a Tuba but it always will result in a huge system that can only be installed in large theatres or by sacrificing ones living environment as can be seen in Figure 10.

5.1.5 Increase of radiating surface

As a less extreme action it is interesting to note that the efficiency can be increased by increasing the diameter of the diaphragm. Equation (9) showed that the radiated sound power is proportional to the radiating surfaces squared:

$$P_\mathrm{a} = x^2_{\mathrm{d,(rms)}}\omega^4 \frac{S^2_\mathrm{d}\rho_0}{2\pi c_0} \quad [\mathrm{W}] \tag{33}$$

For example a factor 2 in radiating surface would give a factor 4 in sound power (=6 dB) in case all other parameters are kept equal. This can be realised by taking a second loudspeaker driven by the same amplifier (or a second amplifier with the same output voltage) in the near vicinity of the first loudspeaker. In that case the amplitude per loudspeaker driver is equal to the situation with one loudspeaker and the two loudspeakers will now deliver together 4 times the output power at 2 times the input power. One might conclude that taking ten loudspeakers would increase the power with a factor hundred and at a certain moment the radiated power would be larger than the consumed power. This seemingly “Perpetuum Mobile” is, however, only true as long as the combined pressure is not (significantly) influencing the amplitude of the movement of the membranes, which is less the case with a large number of loudspeakers. At higher efficiency levels this beneficial effect will decrease asymptotically although it is hardly relevant for practical systems with only a few loudspeakers. An alternative way to explain the increase in efficiency is by considering that the pressure of both loudspeakers add together and the resulting total pressure works on the velocity of both diaphragms to create the sound power that is radiated. This addition of pressure at the diaphragms is only happening when the distance between the loudspeakers is smaller than the wavelength of the sound as otherwise phase differences due to the distance would reduce the total pressure level. This also means that two subwoofers in the same room will act as four, when placed together. In that case one should reduce the low-frequency input signal with 6 dB to compensate the factor four in radiated power, if the system was designed for the use with one subwoofer.

5.1.6 Ultra Low Frequency Efficiency

As mentioned before, subwoofers have to deliver a significant amount of sound at frequencies below the first resonance frequency, where the excursion levels are large. The excursion in this frequency range is strongly determined by the stiffness of the enclosure as was given by Equation (30):

$$k_\mathrm{a} = \lambda S^2_\mathrm{d}\frac{P_0}{V_\mathrm{e}} \quad [\mathrm{N/m}] \tag{34}$$

While the example in the previous section suggested that increasing the radiating surface would increase the efficiency, this was stated under the condition that all other factors would remain unchanged, hence the same excursion level. For frequencies below the first resonance frequency this would imply that a larger radiating

surface would need a squared larger volume of air in the enclosure to obtain the efficiency benefit. Or one would need to take the double volume in case also the input power is doubled, which was the case with the example of taking two identical subwoofers with two amplifiers.
It is interesting to quantify this observation by combining Equation (34) with Equation (33) in a proportionality relation of the radiated sound power to the excursion x, the enclosure volume V_e the radiating surface S_d for a constant actuator force and frequency:

$$\begin{aligned}
P_\mathrm{a} &\propto x^2_{\mathrm{d,(rms)}} S^2_\mathrm{d} \\
x_{\mathrm{d,(rms)}} &\propto \frac{1}{k} \quad , \quad k \propto \frac{S^2_\mathrm{d}}{V_\mathrm{e}} \quad \Longrightarrow x^2_{\mathrm{d,(rms)}} \propto \frac{V^2_\mathrm{e}}{S^4_\mathrm{d}} \\
P_\mathrm{a} &\propto \frac{V^2_\mathrm{e}}{S^2_\mathrm{d}}
\end{aligned} \tag{35}$$

This implies that with the same electrical input power a larger surface would require a proportional larger volume to give the same radiated sound power.
Another conclusion might be that a for a given enclosure volume a smaller radiating surface would increase the radiated sound power and hence the efficiency. Unfortunately that would lead to extreme excursion levels following Equation (33), while a smaller radiating surface would also limit the possibility to generate sufficient force from the actuator.

This all leads to a final conclusion that a loudspeaker for an ultra-low frequency should be selected by following the next steps:

1. The radiating surface should be sufficient to deliver the required sound power with a diaphragm excursion that stays within the "linear-range" as given in the specifications.

2. The amplifier should be able to deliver the maximum electrical power that the loudspeaker can handle in regular music conditions. Often that is a factor 2 above the allowed maximum continuous electrical power.

3. The minimum enclosure volume is determined by the maximum allowable stiffness to realise the required maximum excursion with the maximum available electrical output current and voltage

Regarding the second item it should be noted that at and around the first resonance frequency the impedance of the loudspeaker is much higher than at frequencies below or above the first resonance frequency, resulting in a lower power from the amplifier.